Code: MAT/03 - Geometry **Credits: 9**
Matter: Geometry
Main language of instruction: Italian
Other language of instruction: English

Teaching Staff
Head Instructor:
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Introduction

The course of Geometry is one of the basic courses planned for the first year of the Bachelor degrees in Civil Engineering, Electronic and Computer Engineering, Industrial Engineering.
The program is basically divided into two parts: the first part concerns the study of Linear Algebra, the second part involves the application of the concepts and calculus techniques previously learned to problems of Analytic Geometry, both in the plane and in the space.
In the first part of the course, the basic notions of Linear Algebra are presented: vector spaces $\mathbf{R}^n$, elements of matrix calculus (sum, scalar multiplication, transposition, rank, determinants, inversion). These tools are fundamental for the discussion and resolution of linear systems, for the study of linear operators on $\mathbf{R}^n$, and for their diagonalization.
In the second part of the course, the notions of Linear Algebra and the calculus techniques (rank, determinants, resolution of linear systems) are applied to the study of problems of geometric nature concerning fundamental entities of the Euclidean plane (points, lines, curves) and of the Euclidean space (points, planes, lines, surfaces). The analytical representation of the Euclidean plane and of the Euclidean space, by means of the introduction of an orthonormal Cartesian coordinate system, allows to apply the basic notions of Linear Algebra to the determination of the Cartesian equations of remarkable geometric loci and to the study of their properties.

Objectives

The course of Geometry has the following objectives:
1. illustrate the elementary theory of the vector spaces $\mathbf{R}^n$;
2. illustrate the basics of matrix calculus;
3. illustrate the discussion and resolution of linear systems;
4. illustrate the basic theory of linear operators on $\mathbf{R}^n$;

5. illustrate the fundamental techniques of the Analytic Geometry in the Euclidean plane;
6. illustrate the fundamental techniques of the Analytic Geometry in the Euclidean space.

Competencies:

A. Knowledge and understanding.

At the end of the course, the student should know the algebraic structure of vector space, the basic tools of matrix algebra, the main techniques for solving linear systems, the properties of linear operators, the most important properties of the fundamental geometric entities of the Euclidean plane and of the Euclidean space, and their Cartesian representation.

B. Applying knowledge and understanding.

At the end of the course, the student should be able to solve Linear Algebra problems using matrix calculus: in particular, the matrix formulation of the theory of linear operators will allow the student to apply the techniques of Linear Algebra to the study of operators and their diagonalization. Furthermore, the knowledge of the analytic representation of the plane and of the space will allow the student to apply the basic techniques of Linear Algebra to the determination and study of the Cartesian equations of geometric loci, transforming a problem of geometric nature of simple or medium level into an analytic problem to be solved with algebraic tools.

C. Making judgement.

At the end of the course, the student should be able to identify autonomously the best formulation to adopt for the study of a geometric problem, which will allow him to reach the conclusion in a short time and limiting, as far as possible, the complexity of the computations.

D. Communications skills.

At the end of the course, the student should be able to describe and discuss problems concerning linear operators by introducing an appropriate matrix language, as well as to describe and solve problems concerning Euclidean Geometry of the plane and of the space by translating a geometric problem into a purely analytic language, using terminology and notations at the same time simple and effective.

E. Learning skills.

At the end of the course, the student should have mastered a formal and rigorous language, as well as the basic notions and techniques of Linear Algebra and Analytic Geometry. All this will allow him to continue his Engineering studies with good maturity and autonomy, rigor and logical capacity, making him able to face other courses in the Bachelor program, where on the one hand an ability to understand logical and concrete problems is required, and on the other hand the techniques of matrix calculus find continuous application.

Syllabus

Program of the course:

Subject 1: ALGEBRAIC STRUCTURES

Sets and set operations. Correspondences, functions. Groups. Fields. The vector space $\mathbf{R}^n$. Linear dependence and independence. Vector subspaces. Generators, bases, coordinates. Standard scalar product in $\mathbf{R}^n$. Norm of a vector. Orthogonality. Orthonormal bases of $\mathbf{R}^n$.

Subject 2: MATRICES AND DETERMINANTS

First definitions. Operations: sum and scalar multiplication. Transposition: symmetric and skew-symmetric matrices. Matrix multiplication. Determinant: definition and properties. Adjoint matrix and inverse matrix of a square matrix. Rank of a matrix. Submatrices and minors of a matrix. Row equivalent matrices. Gaussian reduction and applications. Change of basis in $\mathbf{R}^n$.

Subject 3: LINEAR SYSTEMS

Generalities on linear systems. Consistent and inconsistent linear systems. The Rouché-Capelli Theorem. Linear systems of n equations with n variables: the Cramer Theorem. Linear systems of m equations with n variables: normal systems and non normal systems. Linear homogeneous systems. Linear systems and associated homogeneous linear systems. Gauss method.

Subject 4: LINEAR OPERATORS ON $\mathbf{R}^n$

Definition and first properties. Image and kernel. Rank and nullity of a linear operator. Injectivity and surjectivity. Matrices associated with a linear operator on $\mathbf{R}^n$. Isomorphisms. Eigenvalues and eigenvectors. Diagonalizable operators. Similar matrices. Diagonalizable matrices. Symmetric linear operators and their diagonalization. Orthogonal matrices and congruent matrices. Change of orthonormal basis.

Subject 5: ANALYTIC GEOMETRY IN THE PLANE

Orthonormal Cartesian coordinate system: coordinates of a point. Definition of geometric vector. Operations on geometric vectors: sum, scalar multiplication. Cartesian representation of geometric vectors. Parallel vectors. Scalar product. Distance between two points. Midpoint of a line segment. Orthogonal component of a vector with respect to a line. Parametric and Cartesian equations of a line.

Direction parameters. Intersection and parallelism of two lines. Sheaf of lines. Direction cosines. Angle between two lines. Perpendicularity of two lines. Distance between a point and a line. Area of a triangle. Curves and remarkable geometric loci in the plane: circle, ellipse, hyperbola, parabola. Change of orthonormal Cartesian coordinate system.

Subject 6: ANALYTIC GEOMETRY IN THE SPACE

Orthonormal Cartesian coordinate system: coordinates of a point. Definition of geometric vector. Operations on geometric vectors: sum, scalar multiplication. Parallelism and coplanarity of vectors. Scalar product. Vector product. Mixed product. Distance between two points. Parametric and Cartesian equations of a plane. Parallelism of two planes. Sheaf of planes. Parametric and Cartesian equations of a line. Direction parameters. Parallelism of two lines. Parallelism between a line and a plane. Coplanarity of two lines. Direction cosines. Angle between two lines. Perpendicularity of two lines. Angle between a line and a plane. Perpendicularity between a line and a plane. Angle between two planes. Perpendicularity of two planes. Distance between a point and a plane. Distance between a point and a line. Distance between two skew lines. Remarkable surfaces: sphere, ellipsoid, elliptic hyperboloid, hyperbolic hyperboloid, elliptic paraboloid, hyperbolic paraboloid. Change of orthonormal Cartesian coordinate system.

Evaluation system and criteria

Written examination.
Maximum mark: 30/30 cum Laude.
Details:
15/30: Three exercises on Linear Algebra;
13/30: Two exercises on Analytic Geometry;
03/30: Six E-tivities / true-false questions.

Bibliography and resources

1. Materials to consult:

Teaching materials are provided by the Instructor and contain lecture notes, slides and video lessons in which the teacher comments on the slides. These materials offer all the elements necessary to deal with the study of the course (course contents and examination program).

2. Recommended bibliography:

- Serge Lang, Linear Algebra. Undergraduate texts in Mathematics, 1987. Springer.
- Giovanni Landi, Alessandro Zampini, Linear algebra and analytic geometry for physical sciences. Undergraduate Lecture Notes in Physics, 2018. Springer.